Available online at www.sciencedirect.com

ScienceDirect

Procedia Structural Integrity 2 (2016) 2682–2689

Structural Integrity Procedia

www.elsevier.com/locate/procedia

21st European Conference on Fracture, ECF21, 20-24 June 2016, Catania, Italy

A cohesive-zone model for steel beams strengthened with pre-stressed laminates

Stefano Bennati[a], Davide Colonna[a], Paolo S. Valvo[a,]*

[a]University of Pisa, Department of Civil and Industrial Engineering, 56122 Pisa, Italy

Abstract

We analyse the problem of a simply supported steel beam subjected to uniformly distributed load, strengthened with a pre-stressed fibre-reinforced polymer (FRP) laminate. According to the assumed application technology, the laminate is first put into tension, then bonded to the beam lower surface, and finally fixed at both its ends by suitable connections. The beam and laminate are modelled according to classical beam theory. The adhesive is modelled as a cohesive interface with a piecewise linear constitutive law defined over three intervals (elastic response, softening response, debonding). The model is described by a set of differential equations with suitable boundary conditions. An analytical solution to the problem is determined, including explicit expressions for the internal forces and interfacial stresses. For illustration, an IPE 600 steel beam strengthened with a Sika® Carbodur® FRP laminate is considered. First, the elastic limit state load of the unstrengthened beam is determined. Then, the loads corresponding to the elastic limit states in the steel beam, adhesive, and laminate for the strengthened beam are calculated. As a result, the increased elastic limit state load of the strengthened beam is obtained.

Peer-review under responsibility of the Scientific Committee of PCF 2016.

Keywords: Steel beam; FRP strengthening; adhesive; beam theory; cohesive-zone model; analytical solution.

1. Introduction

Fibre-reinforced polymers (FRP) are increasingly used in civil engineering for the strengthening of existing constructions. In such applications, the existing structural elements (made of traditional materials such as, for instance, masonry, wood, concrete, steel, etc.) are strengthened by adhesively bonding FRP laminates onto their

* Corresponding author. Tel.: +39-050-2218223; fax: +39-050-2218201.
E-mail address: p.valvo@ing.unipi.it

Peer review under responsibility of the Scientific Committee of PCF 2016.
10.1016/j.prostr.2016.06.335

external surfaces. The type of fibre, shape, thickness, etc. of the FRP laminate vary according to the type of element to be strengthened and the desired level of structural performance (Bank 2006). For the strengthening of steel structures, carbon fibre reinforced polymers (CFRP) are preferred because of their superior mechanical properties (Zhao and Zhang 2007, Gholami et al. 2013). Furthermore, CFRP laminates can be pre-stressed, which enables more effective use of the composite material, contribution of the strengthening in carrying out the dead load, closure of cracks in concrete (Aslam et al. 2015, Haghani et al. 2015), and increased fatigue life in steel (Ghafoori et al. 2015).

The existing structure and FRP laminate behave as a composite structure with a key role played by the adhesive layer, which transfers the stresses between the bonded elements. As a matter of fact, debonding of the FRP laminate due to high interfacial stresses is a relevant failure mode for this type of interventions. Therefore, a wide number of theoretical and experimental studies have been conducted to achieve reliable and accurate evaluation of such interfacial stresses. Smith and Teng (2001) presented a review of the theoretical models for predicting the interfacial stresses and also developed a solution for strengthened beams in bending. Al-Emrani and Kliger (2006) determined the interfacial shear stresses in beams strengthened with pre-stressed laminates subjected to mid-span concentrated loads. Benachour et al. (2008) extended the previous solutions to distributed loads and multidirectional laminates used as strengthening. All the aforementioned models consider the adhesive layer as an elastic interface to obtain simple closed-form solutions. A more realistic modelling of the adhesive, however, requires the introduction of a non-linear (or piecewise linear) cohesive law for the interfacial stresses (De Lorenzis and Zavarise 2009).

Bennati et al. (2012) used a cohesive-zone model to determine the overall non-linear response of an FRP-strengthened beam in pure bending. In this paper, such model is extended to account for the pre-stressing of the laminate. The beam is considered simply supported and subjected to uniformly distributed load. According to the assumed application technology, the laminate is first put into tension, then bonded to the beam lower surface, and finally fixed at both its ends by suitable connections. The beam and laminate are modelled according to classical beam theory. The adhesive is modelled as a cohesive interface with a piecewise linear constitutive law defined over three intervals (elastic response, softening response, debonding). The model is described by a set of differential equations with suitable boundary conditions. An analytical solution to the problem is determined, including explicit expressions for the internal forces and interfacial stresses. For illustration, an IPE 600 steel beam strengthened with a Sika® Carbodur® FRP laminate is considered. First, the elastic limit state load of the unstrengthened beam is determined according to the Eurocodes (EC 2005). Then, the loads corresponding to the elastic limit states in the steel beam, adhesive, and laminate for the strengthened beam are calculated in line with the Italian regulations on FRP strengthening (CNR 2014). As a result, the increased elastic limit state load of the strengthened beam is obtained.

Lastly, it should be noted that the current model does not take into account the plastic response of the steel beam, which is indeed relevant to the determine the bearing capacity of the system at the ultimate limit state (Linghoff et al. 2010, Linghoff and Al-Emrani 2010). Further studies towards this goal are in progress (Bennati et al. 2016).

2. Mechanical model

Let us consider a steel beam AB of length $2L$, simply supported at its ends and subjected to a uniformly distributed load per unit length, p (as better specified in the following, this load will be a combination of the beam self-weight, g_1, a permanent load due to non-structural elements, g_2, and an imposed load, q). The beam is strengthened by an FRP laminate of length $2l$ adhesively bonded to its bottom surface. As concerns the application technique, we assume that the laminate is first pre-stressed by a suitable axial force, P, then adhesively bonded to the beam, and finally fixed at both its end sections, C and D. We denote with $a = L - l$ the distance of the anchor points from the end sections of the beam (Fig. 1).

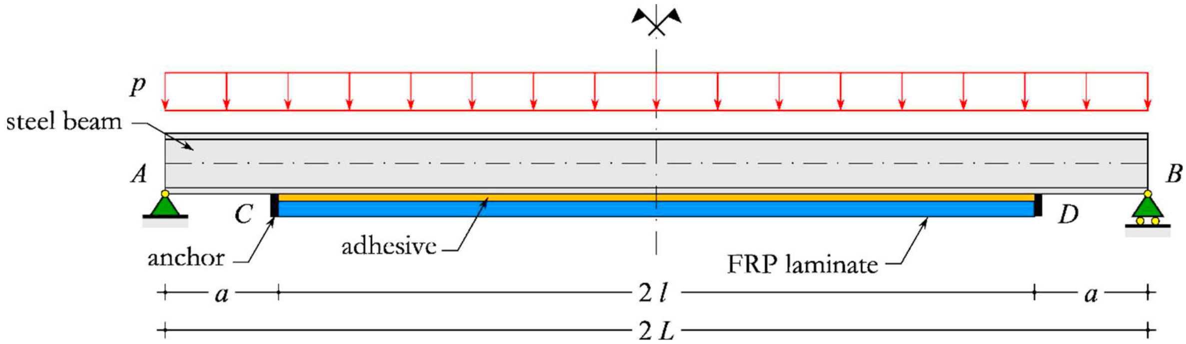

Fig. 1. FRP-strengthened steel beam subjected to uniformly distributed load.

We denote with b_b and h_b respectively the width and height of the beam cross section, with b_f and t_f respectively the width and thickness of the FRP laminate, and with t_a the thickness of the adhesive layer (Fig. 2). Furthermore, we indicate with A_b and I_b the area and moment of inertia of the cross section of the beam, respectively, and with $A_f = b_f\, t_f$ the area of the cross section of the laminate.

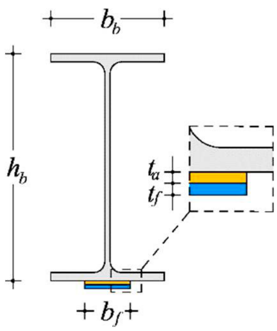

Fig. 2. Cross section of the strengthened beam.

Thanks to the symmetry of the problem, it is possible to limit the analysis to the left-hand half of the system, by introducing appropriate restraints on the axis of symmetry (Fig. 3). The generic cross section of the beam is identified by a curvilinear abscissa, s, measured from the anchor point of the laminate, C. Similarly, the generic cross section of the laminate is identified by a curvilinear abscissa, s^*, also measured from the anchor point, C. We denote with $w_b(s)$ the axial displacements of points at the beam bottom surface and with $w_f(s^*)$ the axial displacements of the laminate cross sections.

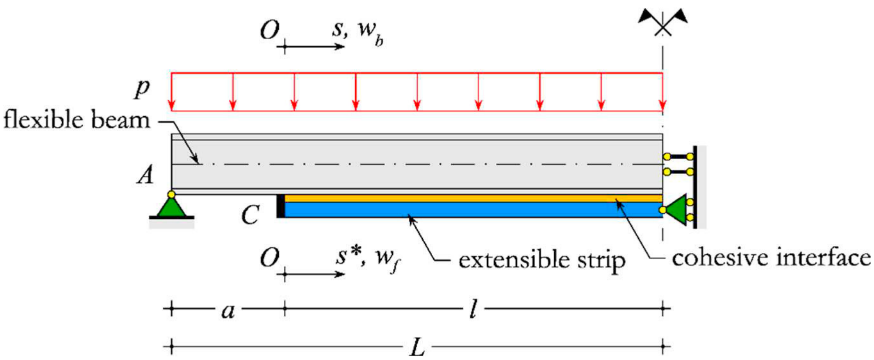

Fig. 3. Mechanical model of the strengthened beam.

In the proposed mechanical model, the steel beam is considered as a flexible beam and the FRP laminate as an extensible strip. An elastic-perfectly plastic behaviour is assumed for steel with Young's modulus E_s and design yield stress f_{yd}. The behaviour of FRP is assumed elastic-brittle with Young's modulus E_f and design tensile strength f_{fd}. The adhesive layer is represented by a zero-thickness cohesive interface, which transfers shear stresses, τ, and no normal stresses. The interfacial stresses depend on the relative displacements, $\Delta w = w_f - w_b$, between the laminate and the bottom surface of the beam. The interface behaviour is considered linearly elastic for shear stresses up to a limit value, τ_0; then, a linear softening stage, corresponding to progressive damage, follows; lastly, debonding occurs. For $\Delta w \geq 0$, the cohesive interface law is given by the following piecewise linear relationship (Fig. 4):

$$\tau(\Delta w) = \begin{cases} k\,\Delta w, & 0 \leq \Delta w \leq \Delta w_0 \quad \text{(elastic response)}, \\ k_s(\Delta w_u - \Delta w), & \Delta w_0 < \Delta w \leq \Delta w_u \quad \text{(softening response)}, \\ 0, & \Delta w_u < \Delta w \quad \text{(debonding)}, \end{cases} \tag{1}$$

where k and k_s are the elastic constants for the elastic and softening responses, respectively; Δw_0 and Δw_u are the relative displacements at the elastic limit and start of debonding, respectively. The elastic constant for the elastic response can be taken as $k = G_a / t_a$, where G_a is the shear modulus of the adhesive.

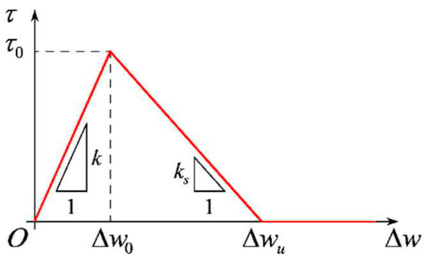

Fig. 4. Cohesive law of the adhesive layer.

3. Structural response

To determine the structural response of the FRP-strengthened beam, it is necessary to distinguish between different stages of behaviour. In what follows, stage 0 refers to the unstrengthened beam, subjected to its self-weight and permanent loads. In stage 1, the laminate is pre-stressed and fixed to the beam. At this point, there is yet no composite action between the beam and laminate, which however are both stressed and deformed because of the dead load and pre-stressing. In the following stage 2, the beam and laminate behave as an elastic composite structure under the imposed loads. This stage ends when either the beam or the adhesive reaches its elastic limit. In stage 3, non-linear response emerges due to plasticity of the steel beam and/or softening of the adhesive layer. Failure of the system occurs when the weakest element (beam, adhesive, or laminate) reaches its ultimate strength.

3.1. Stage 0 – Unstrengthened beam

The unstrengthened beam is subjected to its self-weight, g_1, and permanent load, g_2, both assumed here as uniformly distributed. Such loads will cause both stress and deformation, however within the linearly elastic behaviour regime. At this stage, the axial force, shear force, and bending moment in the beam respectively are

$$N_{b,G}(s) = 0, \quad V_{b,G}(s) = (g_1 + g_2)(l - s), \quad M_{b,G}(s) = \frac{1}{2}(g_1 + g_2)(a + l)(2l + a - s), \tag{2}$$

with $-a \leq s \leq l$. Note that when evaluating ultimate limit states, the loads in Eq. (2) should be suitably factored (EC 2005).

In a real structure, cambering of the beam is often introduced to compensate for the deflection due to dead loads. For simplicity, here we do not consider this and assume the deformed configuration of the unstrengthened beam as the reference configuration for the strains and displacements calculated below (from stage 1 on).

3.2. Stage 1 – Pre-stressing and fixing of the laminate

During the pre-stressing stage, the laminate is put into tension by an axial force, P, applied through the anchor points on the beam bottom surface. Simultaneously, the beam is compressed by the same axial force, which produces also bending because of the eccentricity of the load application point with respect to the beam centreline. The internal forces produced by pre-stressing in the reinforced part of the beam ($0 \le s \le l$) are

$$N_{b,P}(s) = -P, \quad V_{b,P}(s) = 0, \quad M_{b,P}(s) = -\frac{1}{2} P h_b. \tag{3}$$

The internal forces given by Eqs. (3) are to be added to those given by Eqs. (2) to obtain the total internal forces in the beam. All loads should be suitable factored when evaluating ultimate limit states.

Due to pre-stressing, points belonging to the beam bottom surface will move towards the mid-span cross section; conversely, points on the laminate will move towards the anchor point C. From the classic assumption of beam theory, that plane sections remain plane, and the constitutive laws for the beam and laminate, we determine the following (positive) displacement for a point S placed on the beam bottom surface at the abscissa s:

$$w_{b,P}(s) = P\left(\frac{1}{E_s A_b} + \frac{h_b^2}{4 E_s I_b}\right)(l - s); \tag{4}$$

and the following (negative) displacement for a point S^* of the laminate at the abscissa s^*:

$$w_{f,P}(s^*) = -\frac{P}{E_f A_f}(l - s^*). \tag{5}$$

On curing of the adhesive, the beam and laminate behave as a composite structure. To determine the interfacial stresses through Eq. (1), the relative displacements at the interface must be evaluated with respect to the deformed configuration at the end of the pre-stressing stage. To this aim, we consider that points S and S^*, initially not aligned, be placed on the same cross section at the end of the pre-stressing operation (Fig. 5).

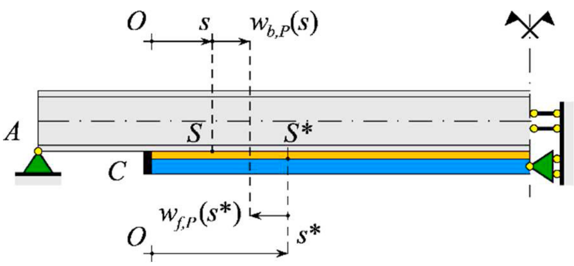

Fig. 5. Displacements of beam and laminate at the end of the pre-stressing stage.

Alignment of points S and S^* after pre-stressing requires that

$$s + w_{b,P}(s) = s^* + w_{f,P}(s^*). \tag{6}$$

By substituting Eqs. (4) and (5) into (6), we determine the relationship between s and s*:

$$s^*(s) = \frac{\left(\frac{h_b^2}{4E_sI_b} + \frac{1}{E_sA_b} + \frac{1}{E_fA_f}\right)l + \left(\frac{1}{P} - \frac{h_b^2}{4E_sI_b} - \frac{1}{E_sA_b}\right)s}{\frac{1}{P} + \frac{1}{E_fA_f}}. \tag{7}$$

3.3. Stage 2 – Application of imposed loads – Linear response

When imposed loads are applied to the strengthened beam, the relative displacement at the interface (with respect to stage 1) turns out to be

$$\Delta w(s) = w_{f,Q}(s^*) - w_{f,P}(s^*) - w_{b,Q}(s) + w_{b,P}(s), \tag{8}$$

where $w_{b,Q}(s)$ and $w_{f,Q}(s^*)$ respectively are the axial displacements of the beam bottom surface and laminate produced by the imposed load, q. In Eq. (8), the abscissa s^* should be calculated through Eq. (7). For $\Delta w \leq \Delta w_0$, the interface behaves elastically, so that Eq. (1) yields the interface shear stress

$$\tau(s) = k\left[w_{f,Q}(s^*) - w_{f,P}(s^*) - w_{b,Q}(s) + w_{b,P}(s)\right]. \tag{9}$$

Figure 6 shows a free-body diagram of an elementary segment of the strengthened beam included between the cross sections at s and $s + ds$. From static equilibrium, the following equations are deduced:

$$\frac{dN_{b,Q}(s)}{ds} = -b_f\tau(s), \quad \frac{dV_{b,Q}(s)}{ds} = -q, \quad \frac{dM_{b,Q}(s)}{ds} = V_{b,Q}(s) - \frac{1}{2}b_fh_b\tau(s), \quad \text{and} \quad N_{f,Q} = -N_{b,Q}, \tag{10}$$

where $N_{b,Q}$, $V_{b,Q}$, and $M_{b,Q}$ respectively are the axial force, shear force, and bending moment in the beam; $N_{f,Q}$ is the axial force in the laminate due to the imposed load.

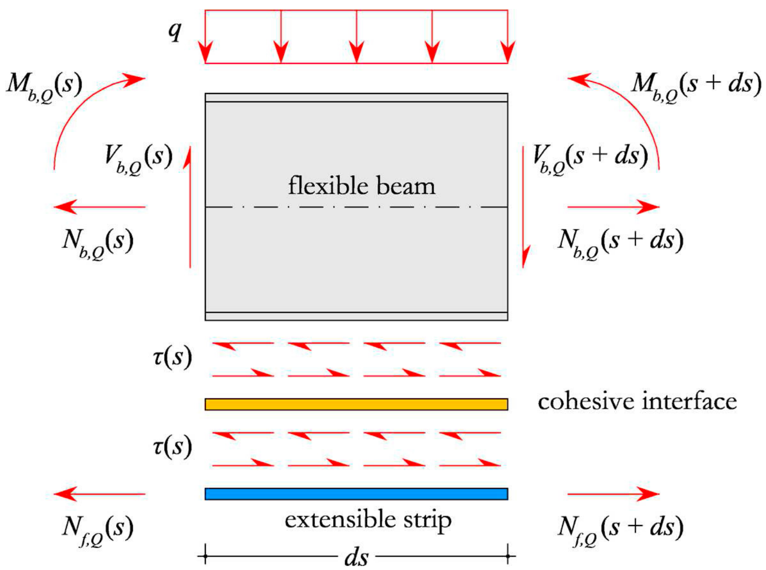

Fig. 6. Free-body diagram of an elementary beam segment.

By solving the differential problem defined by Eqs. (9) and (10) – as explained in detail in Bennati et al. (2016) – the following final expressions are obtained for the interfacial shear stress,

$$\tau(s) \cong \xi q(l-s) - \xi q l \exp(-\lambda s), \tag{11}$$

and internal forces in the beam,

$$\begin{aligned} N_{b,Q}(s) &= \xi q b_f \left\{ s\left(\frac{1}{2}s - l\right) + \frac{l}{\lambda}\left[1 - \exp(-\lambda s)\right] \right\}, \\ V_{b,Q}(s) &= q(l-s), \\ M_{b,Q}(s) &= \frac{1}{2}q(a+s)(2l+a-s) + \frac{h_b}{2}\xi q b_f \left\{ s\left(\frac{1}{2}s - l\right) + \frac{l}{\lambda}\left[1 - \exp(-\lambda s)\right] \right\}. \end{aligned} \tag{12}$$

where

$$\lambda = \sqrt{k b_f \left(\frac{1}{E_s A_b} + \frac{h_b^2}{4 E_s I_b} + \frac{1}{E_f A_f} \right)} \quad \text{and} \quad \xi = \frac{k}{\lambda^2} \frac{h_b}{2 E_s I_b}. \tag{13}$$

3.4. Stage 3 – Non-linear response and failure of the strengthened beam

Stage 3 corresponds to non-linear response due to plasticity of the steel beam or softening of the adhesive. This stage of behaviour cannot be described here because of length limits. See Bennati et al. (2016) for further details.

4. Illustrative example

As an illustration, we consider a S235 steel beam with IPE 600 cross section strengthened with a Sika® Carbodur® CFRP laminate. The material constants, factored according to the Eurocodes (EC 2005) and Italian regulations on FRP strengthening (CNR 2014), and geometric properties used in the example are the following:

- steel: E_s = 210 GPa, f_{yk} = 235 MPa, γ_s = 1.1, $f_{yd} = f_{yk} / \gamma_s$ = 213.64 MPa, A_b = 15600 mm^2, I_b = 920800000 mm^4;
- adhesive: G_a = 4.923 GPa, τ_k = 15 MPa, η_a = 0.85, γ_{fd} = 1.2, $\tau_0 = \eta_a \tau_k / \gamma_{fd}$ = 10.63 MPa, t_a = 1 mm;
- FRP laminate: E_f = 165 GPa, f_{fk} = 2800 MPa, γ_f = 1.1, $f_{fd} = \eta_a f_{fk} / \gamma_f$ = 2163.63 MPa, b_f = 120 mm, t_f = 2.6 mm.

The span of the existing beam, $2L$, and permanent load due to non-structural elements, g_2, are fixed imposing that

$$\begin{cases} \frac{1}{8}(\gamma_{G1} g_1 + \gamma_{G2} g_2)(2L)^2 \le 50\% f_{yd} W_b, \\ \frac{5}{384} \frac{g_1 + g_2}{E_s I_b}(2L)^4 \le \frac{1}{800}(2L), \end{cases} \tag{14}$$

where W_b = 3069000 mm^3 is the elastic section modulus and the partial factors are γ_{G1} = 1.35 and γ_{G2} = 1.50 (EC 2005). By assuming the equal sign in inequalities (14), the maximum theoretical values of $2L$ and g_2 are first determined. Then, by rounding down such values to the nearest integer multiples of 500 mm and 0.50 kN/m, respectively, we obtain $2L$ = 10500 mm and g_2 = 14.50 kN/m.

The imposed load corresponding to the elastic limit state in the unstrengthened beam, q_b^0, is determined from:

$$\frac{1}{8}\left(\gamma_{G1} g_1 + \gamma_{G2} g_2 + \gamma_Q q_b^0\right)(2L)^2 = f_{yd} W_b, \tag{15}$$

and turns out to be 16.14 kN/m. Next, from the following equations, we calculate the value of imposed load corresponding to the elastic limit in the strengthened steel beam, q_b:

$$\pm\frac{1}{8}(\gamma_{G1}g_1+\gamma_{G2}g_2)\frac{(2L)^2}{W_b}+\frac{1}{A_b}\left[\gamma_P N_{b,P}(l)+\gamma_Q N_{b,Q}(l)\right]\pm\frac{1}{W_b}\left[\gamma_P M_{b,P}(l)+\gamma_Q M_{b,Q}(l)\right]=\pm f_{yd}, \tag{16}$$

where the plus and minus signs respectively correspond to the stresses at the lower and upper surface of the beam. Assuming a pre-stressing load $P = 483.6$ kN (50% of mean rupture load), we obtain q_b = 18.67 kN/m, which corresponds to an increase of 15.7% with respect to the unstrengthened beam. Failure of the adhesive and FRP laminate occur for higher values of the imposed load, q_a = 2362.44 kN/m and q_f = 147.85 kN/m, respectively.

5. Conclusions

The developed mechanical model enables determining the increase in the elastic limit load for steel beams strengthened with FRP laminates subjected to uniformly distributed loads. In the shown example, elastic failure of the beam precedes both softening/debonding of the adhesive and rupture of the laminate. Further studies are in progress to evaluate such types of behaviour together with plasticity of the steel beam (Bennati et al. 2016).

Acknowledgements

Financial support from the ERA-NET Plus Infravation 2014 Call within the project SUREBridge (www.surebridge.eu) through subcontract by partner company AICE Consulting Srl (www.aiceconsulting.it) is gratefully acknowledged.

References

Al-Emrani, M., Kliger, R., 2006. Analysis of interfacial shear stresses in beams strengthened with bonded prestressed laminates, Composites Part B, 37 (1996) 265–272.

Aslam, M., Shafigh, P., Jumaat, M.Z., Shah, S.N.R., 2015. Strengthening of RC beams using prestressed fiber reinforced polymers – A review. Construction and Building Materials 82, 235–256.

Bank, L.C., 2006. Composites for Construction, John Wiley & Sons, Hoboken, New Jersey.

Benachour, A., Benyoucef, S., Tounsi, A., Adda Bedia, E.A., 2008. Interfacial stress analysis of steel beams reinforced with bonded prestressed FRP plate. Engineering Structures 30, 3305–3315.

Bennati, S., Colonna, D., Valvo, P.S., 2016. Evaluation of the increased load bearing capacity of steel beams strengthened with pre-stressed FRP laminates. Frattura ed Integrità Strutturale (to be submitted).

Bennati, S., Dardano, N., Valvo, P.S. 2012. A mechanical model for FRP-strengthened beams in bending. Frattura ed Integrità Strutturale 22, 39–55.

CNR 2014. CNR-DT 200 R1/2013, Istruzioni per la Progettazione, l'Esecuzione ed il Controllo di Interventi di Consolidamento Statico mediante l'utilizzo di Compositi Fibrorinforzati (in Italian).

De Lorenzis, L., Zavarise, G., 2009. Cohesive zone modeling of interfacial stresses in plated beams. International Journal of Solids and Structures 46, 4181–4191.

EC 2005. EN 1993-1-1:2005, Eurocode 3: Design of steel structures – Part 1–1: General rules and rules for buildings.

Ghafoori, E., Motavalli, M., Zhao, X.-L., Nussbaumer, A., Fontana, M., 2015. Fatigue design criteria for strengthening metallic beams with bonded CFRP plates. Engineering Structures 101, 542–557.

Gholami, M., Sam, A.R.M., Yatim, J.M., Tahir, M.M., 2013. A review on steel/CFRP strengthening systems focusing environmental performance. Construction and Building Materials 47, 301–310.

Haghani, R., Al-Emrani, M., Kliger, R., 2015. A new method for strengtheng concrete structures using prestressed FRP laminates, ISEC 2015 – 8th International Structural Engineering and Construction Conference: Implementing Innovative Ideas in Structural Engineering and Project Management. Sydney, Australia.

Linghoff, D., Al-Emrani, M., Kliger, R., 2010. Performance of steel beams strengthened with CFRP laminate – Part 1: Laboratory tests. Composites: Part B 41, 509–515.

Linghoff, D., Al-Emrani, M., 2010. Performance of steel beams strengthened with CFRP laminate – Part 2: FE analyses. Composites: Part B 41 (2010) 516–522.

Smith, S.T., Teng, J.G., 2001. Interfacial stresses in plated beams. Engineering Structures 23, 857–871.

Zhao, X.-L., Zhang, L., 2007. State-of-the-art review on FRP strengthened steel structures. Engineering Structures 29, 1808–1823.